The Millikan Oil Drop Experiment

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We used the classical method of Millikan to determine the elementary charge. Small drops of oil falling at constant terminal velocity due to gravity and an applied electric field were observed. The drops accumulated small amounts of electric charge due to friction, this charge accelerated the drop in the electric field. Through measurement of many drops, a discrete value of elementary charge was found to be 1.62±.12 x 10^{-19} *Coulombs. The accepted value of elementary charge, 1.602 176 53(14) x* 10^{-19} *lies well within our error. We conclude that the oil drop experiment does indeed measure the elementary charge, and its value is within the range we measured.*

After the identification of the electron in 1897 by J. J. Thomson, what naturally followed were attempts to measure its properties. Measuring mass without charge proved to be an impossible task, however, one could measure the charge without knowing the mass. In 1909 Robert A. Millikan performed an experiment that successfully measured the quantity of elementary charge, the charge of the electron. The creative design of his experiment showed not only the elementary unit of charge, but also its inherent discreteness.

His experiment utilized the charge build up on oil drops when sprayed in the air. The drops are allowed to enter a region where two plates of opposite voltage create an electric field that forces the drop in a direction depending the sign of the charge. Using classical mechanics, the difference between the speed at which the drop moves with the field on, and the speed at which the drop moves with the field off will be in some proportion to the charge on the drop. Drops are viewed through a microscope lens, placed in between the two plates controlling the electric field. The time it takes to move a certain distance is measured, which yields the velocity of the drop. What makes this experiment so accurate is that the very small oil drops moving in the air move at nearly a constant velocity, their terminal velocity. This is due to the density of the oil or similarly the small mass of the oil drops.

In order to determine the amount of charge per drop based on only the velocity it is moving, the following equations must be used:

$$\textit{Eqn. 1} \qquad q = 3\pi E \sqrt{\frac{9\,(v_f - v_r)}{4\rho' g}}\ \eta^{3/2}\,(v_f + v_r)$$

$$\textit{Eqn. 2} \qquad q = 6\pi E \sqrt{\frac{9\,v_f}{2\rho' g}}\ \eta^{3/2}\,(v_f + v_r)$$

Equations for determining value of charge q. *η = constant based on temperature and pressure of the room, ρ'= oil density – air density, E = electric field, v(f) = falling velocity, v(r) = rise velocity.*

The two equations represent the two methods used in taking data. Eqn. 1 is used in the "forced" fall case, Eqn. 2 is used in the "free" fall case. Obtaining the charge due to the motion of the drop in the field requires only that there is a difference speeds due to a difference in forces.

In the "free" fall case, the drop is allowed to drop with the field turned off, being acted on only by gravity. When the drop reaches a certain point the field is applied such that the drop reverses its direction and moves upwards until it reaches is starting point. This gives us two velocities with two systems of forces.

In the "forced" fall case, the drop is forced down as well as up with the electric field. This dramatically increases the time required for taking data, however, it has some drawbacks as well. In Eqn. 1 the difference of the two velocities is taken, as opposed to Eqn. 2 where only sum is needed. Due to the fact that the velocities are very slow and consequently small numbers, along with the fact that the largest uncertainty in the experiment comes from determining when precisely to switch the motion of the drop, the difference of these values results in possibly flawed data. Taking the difference of two very small numbers with large uncertainties yields a number close to zero, with larger uncertainty. While we were able to get good data using this method, it is best to avoid it if not for any reason other than it is avoidable.

The fundamental idea is that the speed in which the electron moves with the field on relative to the speed it moves with the field off, allows for the determination of the amount of charge; so long as all other values are well known (i.e. electric field strength, distance between plates, etc.) Therefore, the only value we need to directly measure is the time it takes the drop to move between a set distance; this will give us the velocity.

The following data shows the recorded times required for the drop to move a set distance in each direction and their corresponding charge. The data for the free fall model are shown in Table 1, the forced fall data is shown in Table 2.

Table 1. *Data recorded for eight drops using the free fall model and their corresponding charge.*

Drop Number	t Rise (s)	t Fall (s)	Charge (Coulombs)	Uncertainty (Coulombs)
1	7.87	37.98	3.80E-19	3.82E-20
2	17.34	46.80	1.93E-19	5.15E-21
3	13.51	16.79	1.91E-19	1.12E-20
4	10.93	18.97	1.95E-19	1.14E-20
5	13.21	17.61	-5.14E-19	1.34E-20

6	12.36	60.63	-2.12E-19	8.25E-21
7	7.73	21.51	5.28E-19	1.02E-20
8	12.38	51.72	1.99E-19	6.46E-21

Table 2. *Data recorded from thirteen drops using the forced fall model and their corresponding charge.*

Drop Number	t Rise (s)	t Fall (s)	Charge (Coulombs)	Uncertainty (Coulombs)
1	8.85	7.55	2.13E-19	3.44E-20
2	15.66	9.34	1.95E-19	1.29E-20
3	10.89	5.92	3.59E-19	1.73E-20
4	10.14	7.72	-2.05E-19	1.15E-20
5	3.33	2.82	-7.66E-19	7.35E-20
6	3.16	2.82	-6.54E-19	1.22E-19
7	7.43	6.45	-2.14E-19	2.53E-20
8	3.82	3.09	-7.22E-19	7.47E-20
9	8.29	3.49	-8.05E-19	5.42E-20
10	3.20	2.62	-9.09E-19	7.16E-20
11	6.12	4.83	-3.88E-19	2.59E-20
12	8.58	7.14	-2.03E-19	2.66E-20
13	10.00	5.80	-3.63E-19	1.96E-20

To make sense out of this data we take the different charge values associated with each drop and plot them on a graph (Fig. 1).

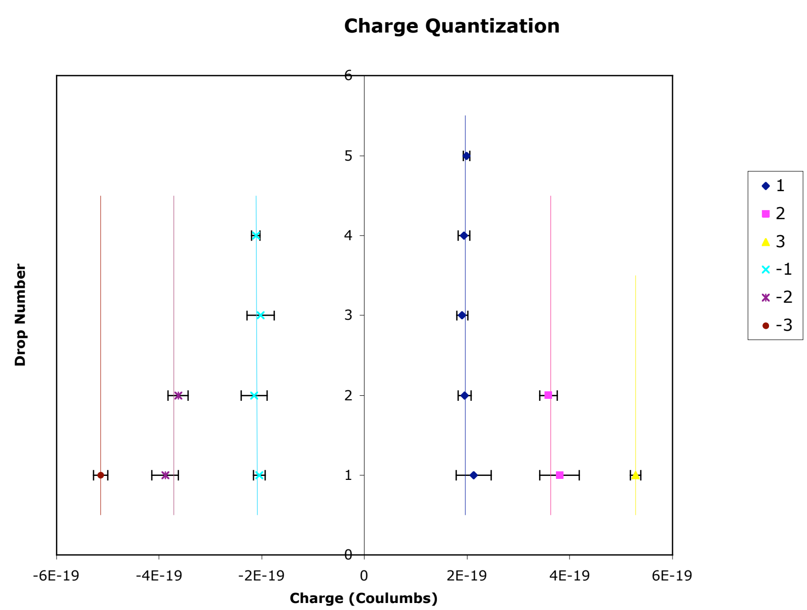

Figure 1. *Plot of charge values. It is evident that there are discrete quantities of charge, especially near the origin.*

Throughout the experiment we were noticing that occasionally the speed of the drop would change in the electric field. When this occurred with a drop with a low amount of charge the change was significant. However, if the drop already had many charges, the difference in speed was unnoticeable and made it into the data. The result is that drops with more charge consequently have larger uncertainties; this is seen clearly in Fig. 1.

We concluded that this was caused the same way the initial drops obtain their charge, through friction between air molecules. As the drops were being pulled back and forth in the field, occasionally the drop would lose or gain an electron. Sometimes if the charge was low enough, the field would no longer affect the drop and we would not be able to pull it back up. Considering that drops with more charge would be less affected by a change of one electron it is expected that this change in total charge would change the speed of the drop and consequently increase its uncertainty. As we increase the amount of charge, it should eventually become useless to draw any conclusions from such data. This is shown in Fig. 1, as the charge gets to nearly five times the charge near the origin, the difference between discrete values is unobtainable and the error bars begin to grow rapidly. In order to gain better perspective on the true value of elementary charge, we chose to examine closely only the values that behaved discretely. The drops with charge less than | 6 x 10^{-19} | Coulombs had this behavior, they are plotted separately in Fig. 2.

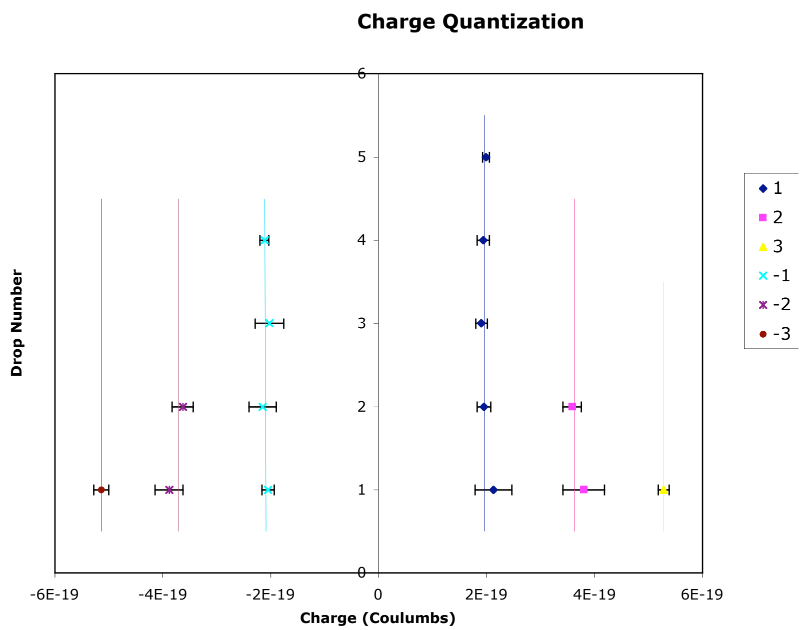

Fig. 2. Plot of charges with lowest uncertainty and consequently exhibit finer charge quantization.

The lines between points in Fig. 2 are placed at the point of the weighted average of each data set. From these values we can get to our final value of the elementary charge. By taking the difference between the total charge of each set, we will come up with a discrete quantity for each jump between integer amounts of electrons. Finally, taking a weighted average of the differences we will come to a value for elementary charge and our uncertainty. Table 3 shows these values along with their uncertainties.

Table 3. Differences in total charge of sets of data with close to the same value.

	Number of e	Difference (Coulombs)	Uncertainty (Coulombs)	Final e (Coulombs)	Final uncertainty of e (Coulombs)
positive	3->2	1.59E-19	2.59E-20	1.62E-19	1.18E-20
	2->1	1.71E-19	2.04E-20		
negative	3->2	1.39E-19	2.9E-20		
	2->1	1.67E-19	2.19E-20		

The advantage to determining the elementary charge value based on the difference between sets of similar values is that errors which propagate throughout the experiment will be removed at the end. As you can see our final value for the elementary charge is significantly smaller than the smallest charge we measured. This offset appears for both positive and negative charges, however, we do not consider the difference between the smallest charge and zero. The errors responsible for offsetting our total charge, in our case increasing the total, exist in all measurements, therefore are removed when we only consider the difference of two points. What we are left with are only the uncertainties in the values measured for q, these uncertainties come from natural errors such as determining when a drop has reached a certain point using only our eyesight. Such errors could be minimized using better instrumentation, as well as taking more data.

The data collected which assigns elementary charge the values of *1.62±.12 E –19* coulombs includes the measured value of *1.5924(17) E-19* coulombs found by Millikan. Today, the accepted value of *e* is *1.602 176 53(14) E-19* coulombs. Both these values lie well within our measured value. Our measurements are accurate to within 98% of Millikan's data, and within nearly 99% of the accepted measurement. From this we can conclude that Millikan's method does provide us with the value of elementary charge varying by no greater than 2%.